Unit 3. Instrumental variables estimation of single-equation linear models

Block D. IV solutions to omitted variables: multiple indicators

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Outline of Block D

1. D. IV solutions to omitted variables: multiple indicators
   Indicators and the multiple indicators solution
   Application: two measures of culture

2. The answer for the Ministry

Indicators of an omitted variable

- Consider the model

$$y = \beta_0 + \beta_1 x_1 + .. + \beta_k x_k + \gamma q + v \quad \text{where } E(v \mid x_1, ...x_k, q) = 0 \tag{45}$$

  and q is unobservable.
- One possible solution to estimate the β_j is to include q into the error term and to find instruments for all the explanatory variables that are potentially correlated with q.
- An alternative solution is to find what are called **indicators** of q.
- Like proxy variables, indicators have to be redundant in the structural equation, that is, an indicator q_1 has to satisfy

$$E(y \mid \mathbf{x}, q, q_1) = E(y \mid \mathbf{x}, q) \tag{46}$$

  where $\mathbf{x} = (x_1, .., x_k)'$.

Indicators vs. proxies

- The key difference with the proxy variable is that the indicator has to satisfy

$$q_1 = \delta_0 + \delta_1 q + a_1$$
$$\text{where } cov(q, a_1) = 0 \text{ and } cov(\mathbf{x}, a_1) = 0 \tag{47}$$

- Whereas the condition for the proxy variable is

$$q = \theta_0 + \theta_1 q_1 + r_1$$
$$\text{where } cov(q_1, r_1) = 0 \text{ and } cov(\mathbf{x}, r_1) = 0 \tag{48}$$

  and these two assumptions ((47) and (48)) are very different.
- The classical errors-in-variable model is an example of an indicator (q_1 will be the observed measure $\delta_0 = 0$ and $\delta_1 = 1$).

The multiple indicators solution

- To use the indicator q_1 we need additional information. For instance, if we have another indicator q_2, that is, if

$$q_2 = \rho_0 + \rho_1 q + a_2$$

where $cov(q, a_2) = 0$ and $cov(x, a_2) = 0$

and

$$cov(a_1, a_2) = 0$$

- Then substituting $q = -\frac{\delta_0}{\delta_1} + \frac{1}{\delta_1} q_1 - \frac{1}{\delta_1} a_1$ in (45) we have

$$\begin{aligned} y &= \beta_0 + \beta_1 x_1 + .. + \beta_k x_k + \gamma(-\frac{\delta_0}{\delta_1} + \frac{1}{\delta_1} q_1 - \frac{1}{\delta_1} a_1) + v \\ &= \alpha_0 + \beta_1 x_1 + .. + \beta_k x_k + \gamma_1 q_1 + v - \gamma_1 a_1 \end{aligned} \tag{49}$$

where

$$\gamma_1 = \frac{\gamma}{\delta_1}$$
$$\alpha_0 = \beta_0 - \frac{\gamma \delta_0}{\delta_1}$$

Why the second indicator is a valid instrument

- Notice that, contrary to the proxy variable approach, q_1 is endogenous in equation (49) and therefore the *OLS* estimator is not consistent.
- However, q_2 can be used as instrument for q_1 in (49) because:
  - q_2 is uncorrelated with v because it is redundant in (45).
  - q_2 is also uncorrelated with a_1 because q and a_2 are uncorrelated with a_1.
  - q_1 is partially correlated with q_2 (through q).
- Of course the roles of q_1 and q_2 can be reversed.
- This solution to the omitted variables problem is called the **multiple indicators solution**.

No lottery? Measure the omitted variable twice

- Most regions will not run a lottery. Can we fix the culture problem without one?
- We have two imperfect measures of culture:
  - `fair_score`: employees' rating of pay fairness (0–10);
  - `women_mgmt`: % of managers who are women.
- Unit 2's answer: use one as a **proxy** and add it as a control. That removes *part* of the bias.

How good are they?

corr(`fair_score`, *culture*)	0.73
corr(`women_mgmt`, *culture*)	0.54
corr(`fair_score`, `women_mgmt`)	0.41

Two noisy measures of the same thing, with *unrelated* noise: survey scales, test scores, sensor readings. This is common in survey and product analytics.

Analysis D1 · Proxy vs. multiple indicators

Coefficient on `transp` *(controls X, robust s.e.)*

Specification	$\widehat{\beta}_1$	s.e.
OLS, no measure of culture	−0.0561	0.0019
Proxy: + `fair_score`	−0.0444	0.0020
Proxy: + both indicators	−0.0415	0.0020
`fair_score` instrumented by `women_mgmt`	**−0.0316**	0.0026
`women_mgmt` instrumented by `fair_score`	**−0.0301**	0.0026
Oracle (culture observed)	−0.0310	0.0020

- The proxies close part of the gap between OLS and the oracle: each indicator is culture *plus noise*, so controlling for it only partly controls for culture.
- Instrumenting one indicator with the other removes the noise problem: the estimate matches the oracle.
- And it is **precise**: s.e. 0.0026, against 0.0098 for 2SLS with the lottery.

This works because here culture is the *only* reason adoption is endogenous, and the two measurement errors are unrelated. If the survey noise were correlated with women in management, or firms adopted for other hidden reasons, only the lottery would still be valid.

Block D in R

```
cor(d$fair_score, d$women_mgmt)

# Proxy approach (Unit 2)
rob(lm(paste("dgap ~ transp + fair_score +", X), d))

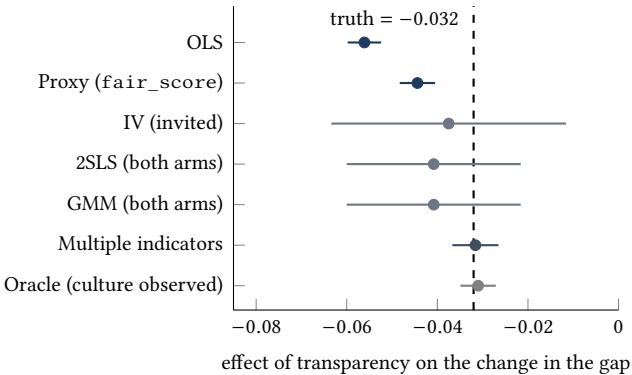
# Multiple indicators
i1 <- ivreg(paste("dgap ~ transp + fair_score +", X,
                  "| transp + women_mgmt +", X),
            data = d)
i2 <- ivreg(paste("dgap ~ transp + women_mgmt +", X,
                  "| transp + fair_score +", X),
            data = d)
rob(i1, "HC0"); rob(i2, "HC0")
summary(i1, diagnostics = TRUE)   # first-stage F
```

1. Why is `transp` not instrumented in these regressions? What are we assuming about it?
2. Include both indicators as regressors, instrumented by nothing. Which assumption fails?
3. Could you add `letter` and `software` as extra instruments? For which regressor?

Summary of indicators

1. An **indicator** measures the omitted variable with error: $q_1 = \delta_0 + \delta_1 q + a_1$.
2. Plugging in one indicator makes it endogenous (its error a_1 enters the equation).
3. A second indicator with $cov(a_1, a_2) = 0$ is a valid instrument for the first.
4. Classical measurement error is a special case, so this also solves errors-in-variables problems.

Every estimator against the truth



Points and 95% confidence intervals (robust).

- OLS and the proxy: precise and **biased**.
- The lottery-based estimators all cover the truth. They are noisier, and 2SLS/GMM with both arms are the most precise of them.
- The indicators match the oracle, *if* their assumptions hold.

What the real evidence says

Study	Finding
Denmark, 2006 law Bennedsen et al. (2022, *JF*)	Gap falls by 2 pp (13%), mainly through slower wage growth for men.
UK, reporting from 2018 Blundell, Duchini, Simion & Turrell (2025, *AEJ: Policy*)	Closes 19% of the gap, by reducing men's pay growth.
Canada, university salary disclosure Baker et al. (2023, *AEJ: Applied*)	Gap falls by about 20–40%.
US, laws protecting pay discussions Cullen & Pakzad-Hurson (2023, *Econometrica*)	Wages fall by about 2%: workers lose bargaining power.

- Our simulated world was built to look like this evidence: a modest effect, mostly through men.
- None of these studies had a lottery. They use **size thresholds** and **timing** of laws, the natural experiments of Block A, with difference-in-differences and discontinuity designs.

Writing the memo

Exercise (10 minutes, in pairs). Write five lines to the Ministry:

1. your estimate of the effect of pay transparency on the gap;
2. its uncertainty;
3. why you trust it more than the raw comparison of adopters and non-adopters;
4. who bears the adjustment (men's or women's wages);
5. one thing that could make it wrong.

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A good memo quotes the lottery-based number with its interval, not the most precise number on the chart.